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These considerations lead at once to the following expressions for the required numbers: 3^5x^6-d, 3^5x^6, 3^5x^6+d, where x, d, are any numbers.

The sum is evidently $(3x)^6=[(3x)^2]^3=[(3x)^3]^2$.

As an example we may take $x=1$, $d=100$.

$$143,\ 243,\ 343,\ \text{whose sum}=729=27^2=9^3.$$

II. **Solution by the PROPOSER.**

Let $\frac{1}{3}(x-y)^2$, $\frac{1}{3}(x^2+y^2)$, $\frac{1}{3}(x+y)^2$ be the three numbers.

Their sum is x^2+y^2.

Let $x^2+y^2=a^6m^6$. Let $x=m^2-n^2$, $y=2mn$.

$\therefore x^2+y^2=(m^2+n^2)^2=a^6m^6$.

$\therefore m^2+n^2=a^3m^3$.

Let $n=pm$.

$\therefore m^2(1+p^2)=a^3m^3$.

$\therefore m=\dfrac{1+p^2}{a^3}$, $n=\dfrac{p(1+p^2)}{a^3}$.

$$x=\frac{(1+p^2)^2(1-p^2)}{a^6},\quad y=\frac{2p(1+p^2)^2}{a^6},\quad x^2+y^2=\frac{(1+p^2)^6}{a^{12}}.$$

$\therefore$ The numbers are

$$\tfrac{1}{3}\left(\frac{(1+p^2)^2(1-2p-p^2)}{a^6}\right)^2,\quad \tfrac{1}{3}\left(\frac{(1+p^2)^6}{a^{12}}\right),\quad \tfrac{1}{3}\left(\frac{(1+p^2)^2(1+2p-p^2)}{a^6}\right)^2.$$

Also solved by *CHAS. C. CROSS*, *JOSIAH H. DRUMMOND*, *M. A. GRUBER*, *J. SCHEFFER*, and *ELMER SCHUYLER*.

84. **Proposed by the late SYLVESTER ROBINS, North Branch Depot, N. J.**

The nth term of an infinite series of "nests" contains all the prime, integral, rational parallelopipeds that have 3^n for their solid diagonals. It is required to determine the general expression for N=the number of such solids in nth term, and to exhibit the dimensions of all the "eggs" in the first six nests.

Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

In parallelopipeds the square of the solid diagonal equals the sum of the squares of the three dimensions (length, breadth, and height). Whence, $(3^n)^2=$ the sum of three squares.

When $(3^n)^2$=the sum of three *integral* squares, I have found, by inspection, that the entire number of sets of three squares is, in terms of n, $\dfrac{3^n+2n-1}{4}$, of which $\dfrac{3^{n-1}+1}{2}$ are prime sets, and $\dfrac{3^{n-1}+2n-3}{4}$ multiple sets.

$\therefore$ According to the problem, $n=\dfrac{3^{n-1}+1}{2}$.

In *prime* sets of the sum of three squares equal to a square, it will be observed that two of the three squares are even and the other odd.

By means of an extensive table containing the odd numbers that are equal to the sum of two squares, I have found the following complete sets of "eggs" for each nest.

In nest 1, where $n=1$ and $N=1$: 1, 2, 2. PROOF.—$1^2+2^2+2^2=(3^1)^2=9$.

In nest 2, where $n=2$ and $N=2$: 1, 4, 8; 7, 4, 4.

In nest 3, where $n=3$ and $N=5$: 7, 2, 26; 7, 14, 22; 23, 2, 14; 23, 10, 10; 25, 2, 10.

In nest 4, where $n=4$ and $N=14$:

1, 28, 76; 1, 44, 68; 17, 56, 56; 23, 16, 76; 23, 44, 64; 41, 16, 68; 41, 28, 64; 47, 16, 64; 49, 8, 64; 49, 32, 56; 55, 20, 56; 55, 40, 44; 65, 20, 44; 79, 8, 16.

In nest 5, where $n=5$ and $N=41$:

1, 22, 242; 17, 14, 242; 17, 46, 238; 17, 106, 218; 17, 134, 202; 31, 38, 238; 31, 158, 182; 47, 14, 238; 47, 154, 182; 49, 2, 238; 49, 158, 178; 95, 118, 190; 95, 50, 218; 95, 130, 182; 97, 46, 218; 97, 62, 214; 97, 94, 202; 97, 134, 178; 113, 22, 214; 113, 62, 206; 113, 74, 202; 113, 146, 158; 127, 22, 206; 127, 46, 202; 127, 106, 178; 127, 134, 158; 143, 50, 190; 143, 74, 182; 143, 122, 154; 145, 70, 182; 161, 2, 182; 161, 38, 178; 193, 22, 146; 193, 62, 134; 193, 70, 130; 209, 22, 122; 209, 38, 118; 223, 22, 94; 223, 62, 74; 239, 22, 38; 241, 22, 22.

In nest 6, where $n=6$ and $N=122$:

17, 136, 716; 17, 436, 584; 49, 236, 668; 49, 224, 692; 49, 128, 716; 49, 496, 532; 55, 196, 700; 73, 116, 716; 73, 44, 724; 79, 32, 724; 79, 172, 704; 79, 112, 716; 79, 308, 656; 79, 340, 640; 79, 460, 560; 89, 224, 688; 89, 416, 592; 103, 376, 616; 103, 424, 584; 119, 68, 716; 119, 196, 692; 119, 436, 572; 119, 484, 532; 137, 4, 716; 137, 236, 676; 143, 124, 704; 143, 284, 656; 175, 196, 680; 175, 104, 700; 185, 40, 704; 185, 296, 640; 199, 136, 688; 199, 248, 656; 199, 304, 632; 199, 376, 592; 239, 308, 616; 241, 4, 688; 241, 416, 548; 241, 128, 676; 241, 464, 508; 247, 116, 676; 247, 236, 644; 271, 32, 676; 271, 220, 640; 271, 208, 644; 271, 380, 560; 313, 56, 656; 313, 304, 584; 329, 92, 644; 329, 460, 460; 337, 56, 644; 337, 196, 616; 359, 56, 632; 359, 152, 616; 359, 248, 584; 359, 424, 472; 367, 236, 584; 367, 296, 556; 401, 172, 584; 401, 248, 556; 401, 296, 532; 401, 364, 488; 409, 376, 472; 409, 152, 584; 431, 68, 584; 431, 136, 572; 431, 296, 508; 431, 376, 452; 439, 172, 556; 439, 196, 548; 439, 236, 532; 439, 284, 508; 455, 104, 560; 455, 280, 496; 457, 116, 556; 457, 364, 436; 473, 304, 464; 497, 196, 496; 497, 224, 484; 521, 44, 508; 521, 100, 500; 521, 220, 460; 521, 236, 452; 521, 340, 380; 527, 196, 464; 527; 284, 516; 529, 40, 500; 529, 116, 488; 529, 200, 460; 529, 268, 424; 529, 288, 436; 529, 332, 376; 551, 112, 464; 551, 304, 368; 583, 136, 416; 583, 224, 376; 593, 4, 424; 593, 196, 376; 623, 44, 376; 623, 104, 364; 623, 236, 296; 625, 196, 320; 625, 220, 304; 631, 28, 364; 631, 196, 308; 649, 4, 332; 649, 124, 308; 649, 172, 284; 649, 196, 268; 655, 4, 320; 655, 100, 304; 655, 40, 296; 655, 104, 280; 689, 32, 236; 689, 116, 208; 695, 4, 220; 695, 100, 196; 719, 32, 116; 719, 44, 112; 721, 28, 104; 721, 40, 100; 721, 56, 92.

In order to obtain the multiple sets in the several nests, we multiply the prime sets of all the preceding nests by the necessary power of 3.

The multiple sets in nest 4 are 3^3 times the prime set of nest 1, 3^2 times the prime sets of nest 2, and 3 times those of nest 3, making 8 in all.

Also solved by *CHARLES C. CROSS.*

AVERAGE AND PROBABILITY.

56. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics in Drury College, Springfield, Mo.

Find the chance that the center of gravity of a triangle lies inside the triangle formed by three points taken at random within the triangle. [From *Williamson's Integral Calculus.*]